

INTERNATIONAL CONFERENCE ON NEW HORIZONS IN EDUCATION
INTE2012

Investigating the relationship between musical training
and mathematical thinking in children

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Abstract

This study examines the potential for music education to enhance children's mathematical thinking. Specification of potential cognitive correlates between musical and mathematical components is sought and underpins the design (3 variables x 2 conditions each = 6 groups). Nearly 200 children aged 7-8 years experienced weekly music lessons (duration = 9 months). Lessons emphasized melody, rhythm or form; in half of the classes, the teacher made the musical-mathematical parallels explicit.

Apart from the specific musical-mathematical foci, the lesson content was kept as constant as possible within primary school settings. Pre-tests and post-tests in musical, creative, spatial and mathematical thinking were administered. Statistical analyses will examine improvement over time while considering differences among three musical components and two conditions for each.

This research addresses concerns that governments' quests for higher standards in mathematics may result in impoverished curricula with limited access to the arts. If it is shown that musical training appears to benefit logical thinking, as hypothesized, it may add to a growing body of research suggesting that policy-makers and educationalists reconsider curriculum balance.

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Keywords: music; music education; mathematics; mathematical thinking; mathematics education; cognitive correlates; spatial reasoning; spatial-temporal reasoning; Mozart Effect; Makam Effect; Ellington Effect

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1. Introduction

The relationship between music and mathematics has been considered for millennia. Enquiries have emerged in cognitive science and educational research literature in particular over the past four decades. Why should such seemingly different domains be linked? What aspects of both are connected? Can musical training improve mathematical ability? The aim of this study is to add to the body of literature that attempts to answer these questions. In essence, this study seeks improvements in the qualities and skills that can support children's development in mathematics as concomitants of an extended course of musical training focused principally on singing.

Motivation for this study was inspired by research co-conducted by the author at the Center for Arts Education Research at Columbia University. Research at five schools for over six years revealed consistently larger improvements in standardized test scores, particularly in mathematics, by children who had studied violin at a young age in comparison to control groups in the same grade level both within the same school and beyond (Abeles & Sanders, 2006).

1.1. Linking Musical Exposure to Spatial Sense

The modern discourse regarding the primary research question here perhaps began when Rauscher and colleagues (Rauscher, Shaw, Levine, Ky & Wright, 1994) conducted an experiment with university students in which $\sim 1/3^{\text{rd}}$ of the matched groups listened to Mozart's Sonata K. 448 in D Major for 10 minutes and then took spatial reasoning tests using the Stanford-Binet Intelligence Scale. Another group listened to relaxation instructions and the other, nothing. Those who had listened to Mozart ($N=36$) scored 8-9 IQ points higher than the other groups; the effect lasted 15 minutes. Thus was born the phrase, "The Mozart Effect," which implies that listening to Mozart leads to higher intelligence. The findings suggest a causal relationship, specifically in spatial-temporal reasoning, not necessarily general intelligence. Yet this study should be noted with temperance due to a tendency for the findings to be exaggerated. This has occurred, sparking a commercial frenzy with claims that Mozart's music will create "Baby Einsteins." Numerous replications of the study were later conducted. In 1999, Chabris analyzed 15 of such studies and concluded that the findings were not statistically significant. However, Rauscher criticized his report, noting that certain studies should not have been included in Chabris' analysis since they had tested for general intelligence, not specifically spatial.

Hetland (2000a) conducted two meta-analyses of 36 experiments with 2,469 subjects and compared tasks that qualified as spatial-temporal (31 of 36) to other types of spatial measures (5 of 36). Contrast analysis shows that the average effect of the experiments using spatial-temporal measures alone is $r = .20$. Experiments employing only nonspatial-temporal measures produced an average effect of $r = .04$, and experiments that employed a combination of spatial-temporal and nonspatial-temporal measures showed an intermediate effect size ($r = .15$). Hetland's analyses substantiate the idea that the consequence of listening to complex music is specific to spatial-temporal thinking, more than to general spatial thinking and even more than to general intelligence.

This specificity argues against the hypothesis that arousal and mood were the causes of the enhanced spatial reasoning abilities demonstrated (Hussain, Thompson & Schellenberg, 2002). Also helping to rule out the arousal and mood hypothesis were studies that used other forms of music such as electronic dance music, which did not have the same effect. Though the dance music had an impact on levels of arousal and mood, spatial-temporal skills were significantly lower for subjects who listened to this versus the classical music. The highly organized and sequentially evolving nature of Mozart's music may explain part of this effect. Repetition of logical patterns exists within the course of multiple melodic and harmonic modulations and unexpected rhythmic and textural shifts within the overall symmetrical design. Whereas, the patterns within the popular dance music would normally be much simpler and more statically repetitive. Additionally, when subjects were asked to focus on Mozart's music, their spatial reasoning scores were higher, again suggesting that specific elements within the music itself perhaps affected the outcome.

Listening to other forms of music such as jazz or non-western music such as Indonesian gamelan music, Turkish makam or Indian raga could presumably have similar outcomes to classical music of this nature if the particular pieces are similarly complex. Interlocking rhythms and/or melodies of Indonesian, Indian, Arabic, Latin or African music are frequently comprised of multiple layers that can be highly intricate. It would be interesting to see the cognitive effects after using one or more of these types of music while including multi-layered melodic and harmonic elements of these cultures as well. Perhaps it could then be just as appropriate to refer to this phenomenon as the Ellington Effect[†] or the Makam Effect[‡].

Shortly thereafter, Rauscher and colleagues responded to challenges to go beyond listening by conducting studies that looked at the effects of training, particularly with preschool children. One study yielding statistically significant increases in spatial reasoning ability in students after months of training involved keyboard lessons and computer lessons (Rauscher, Shaw, Levine, Wright, Dennis, & Newcomb, 1997). Other responses to Rauscher and colleagues' research include replications (Vaughn, 2000), clarifications (Rauscher, Shaw, & Ky, 1995), enquiries investigating why the effect occurs (Rideout & Laubach, 1996) and attempts to generalize to other cognitive abilities (Wilson & Brown, 1997).

In an effort to thoroughly examine all of these studies, Hetland conducted meta-analyses of 19 experiments involving musical training in a variety of instruments with children ranging in age from 3-12 that met certain criteria, such as having at least one control. She implemented three meta-analyses based on outcome measurements. In line with evidence from the listening studies, categorizations were based on which spatial reasoning tests were employed in the studies. The first, which included 15 (N = 704) studies that utilized spatial-temporal tests, yielded a large average effect size by meta-analysis standards ($r = .37$, $d = .79$). The second meta-analysis included five (N = 694) studies employing nonspatial-temporal measures yielded a small average effect size of $r = .08$, $d = .16$. The third included nine studies (N = 655) that employed a variety of spatial tests (including spatial-temporal). The average effect was moderate but still relatively strong ($r = .26$, $d = .55$), indicating that spatial reasoning skills in general are enhanced via musical training but not as significantly as spatial-temporal skills are ($r = .37$, $d = .79$).

1.2. *Spatial Sense and Musical Processing*

As discussed, numerous studies have shown a connection between music and spatial-temporal reasoning (Rauscher, et al, 1994; Graziano, Peterson, & Shaw, 1999). Since music is sound moving in time, it seems logical that training in this domain could facilitate temporal reasoning, yet why does spatial reasoning also seem to improve with exposure to and particularly with training in music? A recent study by Spelke (2008) offers promising clues to help answer this question.

In her previous and extensive research on mathematical cognition and development, Spelke found that mathematical ability is not confined to one system in the brain. She had also been intrigued by the longstanding idea that there is a special tie between music and mathematics. Thus, when challenged to see how the arts relate to the organization of cognitive systems, she and members of her lab began a correlative study among three school age groups to see if those with music training had an associated advantage in any area of mathematical aptitude. The three studies conducted looked at levels of training from low intensity to high. Of three core mathematical abilities, the children who received moderate or intensive music training performed significantly higher on geometrical and spatial tasks. There was also

[†] This particular usage of the term, "Ellington Effect" has been generated by the author for this article as both a counter and a compliment to the term, "Mozart Effect" to emphasize that music need not be of a particular style in order to cause a certain cognitive effect. It is believed by the author that the precondition is in the complexity and cohesiveness of the music, not the cultural origins. This term has been used in the past by Billy Strayhorn, a cohort of Ellington's, who once called Ellington's ability to create a unique sound via his compositions and arrangements for specific band members the "Ellington Effect" (Tucker, 1993).

[‡] Makam (from the Arabic word مقام), in Turkish classical music, is a system of melody types, which contains a complex set of rules for composing and performance. These rules, however, are open for individual interpretation by the composer and performer (Beken & Signell, 2006).

an associated advantage for the music group in using number lines and maps, which utilize spatial skills as well, even when controlling for elements such as reading IQ and motivation.

In order to discover the source of these correlations, Spelke conducted an infant experiment, patterned after one that her colleagues (Carey & Srinivasan, 2008) conducted. This looked at note durations and visual objects (worms) of different lengths to see if there is an inherent perceptual relationship between musical time and visual space. They had presented infants with short and long note durations accompanied by corresponding short and long worms. The infants readily learned to connect the relationship. To test whether or not this was arbitrary, they presented another group of babies with the same sets of tones and worms, yet they were reverse-paired. Those infants did not learn the connection, suggesting a cognitive relationship between auditory duration and visual length that could reveal a foundational link between the perception of sound and the representation of space.

Spelke's experiment looked at pitch contour in relation to space, in this case height. Rising or descending sequences of tones were matched with corresponding heights of objects, and then reverse pairs were shown to a different group of babies. Again, the infants learned the relationship with the congruent pairs but did not learn to match the incongruent pairs, showing a connection between melodic contours and positions in space. This study suggests that an inherent relationship exists between musical and spatial processing; both may serve as a foundation for an emergence of the positive relationship between music and mathematics.

1.3. Spatial skills & numerical skills: Comparisons with musical thinking

In order to probe further into the reasons for the link between the two domains in discussion, it may be helpful to look at how children's mathematical thinking develops. Children continually attempt to organize their world by finding patterns and creating structures (Gopnik, 2004). Mathematics is an activity of organization, of problem-solving. Organizing subject matter within reality must be accomplished according to mathematical patterns in order to find solutions (Freudenthal, 1991). Music-making also requires the organization of material, and like a mathematician, the musician seeks patterns, creates structures and solves problems (Pogonowski, 1987; du Sautoy, 2007).

In agreement with van Nes and de Lange (2007), one could define a pattern as a numerical or spatial regularity and the relationship between the elements of a pattern as its structure. These researchers give examples of spatial structures that young children would normally be familiar with such as the dot configurations on dice, beads on a necklace and block constructions. If sound is considered in terms of space and patterns, a musical piece is a spatial structure made up of patterns of sound. Additionally, its standard notation is in fact a spatial structure with patterns of curves, dots and lines.

Van Nes and de Lange (p. 217) suggest that the intertwinement of different components of early spatial sense may contribute to the development of children's number sense, the discernment of quantities and relationships between numbers. In parallel, since every sound in music is spatial in some form, whether it is rhythmic, melodic, harmonic or tonal, it can be attached to a number. All of these musical components are in a specific, measurable relationship to the others within its own category as well as between categories. An understanding of these spatial-temporal and numerical elements within a piece may at least contribute to an implicit understanding of the structure of a musical composition as well as the patterns within this structure. Therefore, these spatial-numerical connections within music may help explain the potential link between musical and mathematical understandings.

Once children can imagine a spatial or temporal structure, whether visually or aurally, of a certain number of objects or sounds that are to be maneuvered, the emerging number sense, which includes knowledge of quantities as well as counting, should be largely clarified and strengthened.

Certain musical pieces are particularly effective for encouraging spatial-temporal reasoning as well as counting and accurate quantity discernment. The song *America*, for instance (Bernstein & Sondheim, 1957) is in a mixed meter, with alternating 6/8 and 3/4 time signatures or rhythm patterns. It provides a good opportunity to teach and learn grouping and counting skills and to alert students to the role that rhythm can play in structure as well as in more intangible ways, such as emotional response. See Figure 1 below.

Figure 1. Below the standard method of notation shown at the top are two ways to count the song “America.” Though the underlying tempo and pulse remain constant, the accents (in bold) fall on different beats as reflected by the time signature of 6/8 3/4 and are thus grouped and counted accordingly. The top row of numbers is a standard way to count in line with the underlying eighth note (or quaver) pulses within this alternating time signature. The first measure (or bar) could also be counted as two large beats, each containing three small beats within (compound duple meter, or compound time). If considered in that way, one could count, “1 ee uh, 2 ee uh, 1 &, 2 &, 3 &.” The bottom row shows an alternate way to count which points out the accents and subsequent grouping structure that is accessible to those who are not familiar with standard notation. The top method of counting is appropriate primarily for those who are learning more complex rhythm notations. This musical example is effective for teaching pattern recognition as well as the skills of counting and keeping a steady beat while accentuating certain ones. The challenge of working with the alternating patterns here also provides an opportunity to develop executive cognitive functions such as sustained attention and cognitive flexibility.



1	2	3	4	5	6	1	&	2	&	3	&
1	2	3	1	2	3	1	2	1	2	1	2

Mathematical abilities such as ordering, comparing, generalizing and classifying are supported by an ability to grasp spatial structure (Waters, 2004; NCTM, 2010). More formal, complex operations such as addition, subtraction, multiplication and use of algebraic variables, also benefit from a solid foundation in spatial reasoning (Kieran, 2004; van Nes & de Lange, 2007). Recent research has shown that children with serious mathematical difficulties tend to use minimal levels of structure if at all (Mulligan, Mitchelmore & Prescott, 2005). Thus, it seems clear that improving spatial reasoning is important for mathematical development.

1.4. Music Education in the Schools

Music may indeed provide a method for assisting growth in the area of spatial-temporal reasoning and numerical ability as discussed. This suggests possibilities for a comprehensive education that includes an inter-disciplinary approach for the enhancement of spatial learning skills. It would not be appropriate to simply substitute music training for spatial awareness guidance in the classroom, but it appears to be potentially helpful as an educational supplement for improving children’s mathematical understandings. A growing body of evidence, including Hetland’s meta-analyses (2000b), confirms the assumption that students improve more in both near and far transfer domains through individual lessons and when learning standard notation. Yet revelations of improvement under all conditions are strong enough to encourage the inclusion of music education in schools regardless of the level of privacy or musical reading skill taught. This consideration motivates the research described below. Final results are still to be analyzed.

2. Design and Methods

2.1. Overall Design

In order to test if musical training would improve mathematical thinking and further, to test if differing pedagogical emphases on specific musical elements would enhance specific mathematical correlates, an original between-groups, pretest-treatment-posttest 3x2 multivariate design was created (see Figure 2 below). Additionally, the author wanted to see if there would be a different learning outcome if mathematical links were made explicit in the lessons. Consequently, there are two independent variables: *musical training* (with

three levels that reflect emphases – rhythm, pitch or structure/form) and *mathematical content explicitness* (with two levels – explicit or implicit). There are four dependent variables as measured by standardized tests: 1) *musical ability*, 2) *mathematical skills* for Year 3, as well as two cognitive constructs considered to influence analytical mathematical thinking, which are 3) *spatial-temporal reasoning* and 4) *creativity*. The chart on the following page illustrates the research design.

Figure 2. Below is the research design in terms of groups and the pretest-treatment-posttest process.

Group 1 Measurement → (Pretest)	Intervention → (Rhythm - Explicit)	Group 1 Measurement (Posttest)
Group 2 Measurement →	Intervention → (Rhythm - Implicit)	Group 2 Measurement
Group 3 Measurement →	Intervention → (Pitch Relationships - Explicit)	Group 3 Measurement
Group 4 Measurement →	Intervention → (Pitch Relationships - Implicit)	Group 4 Measurement
Group 5 Measurement →	Intervention → (Structure/Form - Explicit)	Group 5 Measurement
Group 6 Measurement →	Intervention → (Structure/Form - Implicit)	Group 6 Measurement

2.2. Participants

Participants, ages 7-8 (Year 3) of low-mid socioeconomic levels, were chosen from five similar state-funded schools. Out of these five schools, six groups have been identified for this experiment. Two of the schools had two classes of ~30 each and two of the smaller schools were combined into one group. Therefore, each group comprised a self-contained class except for one, which comprised the two smallest classes (Group N = 25-35).

2.3. Activities within groups

The same teacher taught and conducted activities for all groups for consistency, quality and internal validity. Activities were compatible with the English National Curriculum for music in the schools (2010). Lessons were videotaped periodically, with parental permission, as well as the children's consent, for additional validity, monitoring and reference purposes.

All children received weekly 40-minute group singing lessons with additional musical activities included; the activities varied slightly depending upon the specific group. The lessons consisted of vocal technique fundamentals, discussion of ideas (plus composing for structure group), learning/singing songs, playing percussion instruments (pitched or unpitched depending upon group) and some movement, thus building up to a repertoire of eleven pieces over the ~nine months of study. All activities mentioned here were not necessarily done in one lesson.

The basic curriculum was presented to the school and parents before the start of the program. All material was appropriate for this age group and featured music from around the world – the first piece was a greeting song, which included “hello” or “good day” in eight languages. The other ten songs were all from different countries (although the structure group composed one piece, thus technically having two from England). The repertoire was selected in order to broaden the children’s musical (and thus cognitive), cultural and linguistic perspectives. This author also wanted to illustrate that any cognitive development beyond normal growth would not have been due to exposure to “western art” music in particular.

Nine of the eleven songs were the same for all groups in order to keep all factors as equal as possible. Yet, the arrangements, instrumental elements and/or class discussions of the pieces varied depending upon the musical emphasis for the group. The final two pieces for each group were of the same level of challenge as the corresponding non-identical songs of the other groups and were either arranged or discussed according to the musical emphasis of the group or were inherently relevant to the musical emphasis. For example, the melody groups learned a Swiss yodeling song, while the rhythm groups learned a Scottish jig-like song. Both are folk tunes of a similar level of difficulty, the first challenges melodic awareness with sudden leaps in pitch while the second challenges rhythmic awareness particularly when including corresponding clapping and knee bending in time to the beat.

The words and music of all songs were frequently illuminated on the classroom whiteboard while the pupils learned the pieces; two weeks before the concert, they were given the lyrics to take home for additional motivation and re-enforcement. At this point, many of the children had memorized most of the lyrics. At the concert, the students sang all eleven pieces from memory (or at least it appeared so for most), which was noted by two of the head teachers after their performances. Another head teacher noted the authenticity of different cultural representations, notably the African and Japanese pieces.

It is possible that memorization of lyrics may have aided in the children’s cognitive development to the degree that certain mathematical questions on the posttests requiring memory may have improved. Thus, though beneficial to the children, the language component could be a confounding factor in this experiment. On the other hand, evidence showing this potential benefit could support the use of singing in school curricula. In addition to memorizing lyrics and learning words and/or phrases of other languages, the students also memorized the specific melodic and rhythmic patterns of the music itself as well as overall structural components. Therefore, it could be argued that whether or not lyrics are involved, memory development can be an additional benefit of musical training and another reason why the link in question exists since most mathematical computations utilize some degree of memory, often both long-term and working. One might assume that both the memorization of lyrics and the music itself may have improved the students’ memory capacity and usability, and even though it would be difficult to identify to what degree each had an influence, particular mathematical questions that may have benefitted most from memory development will be noted in the analysis.

The songs selected were simple enough for children yet challenging enough to encourage growth as well as the state of “flow” (Csikszentmihalyi, 1988). Flow has been described as the mental state that arises when fully immersed in an activity. Custodero, 1998, describes this phenomenon as occurring while the balance of enjoyment and challenge is optimum.

2.5. *Data collection and analysis*

SAT tests had been administered at the end of the school year before the experiment (at the end of Key Stage 1) and again at the end of that school year; these will be included in the final analyses. Paired sample t-tests will be used when comparing pretests and posttests within each group and two-way independent analysis of variance (ANOVA) when comparing the groups to each other and exploring their interrelationships. Two-way independent ANOVA will be used because two independent variables are being measured and each of the different participants will be contributing one score to the data (different participants took part in all conditions). However, though the six groups have been matched as closely as

possible within the natural schooling system, in order to control for initial differences between groups and to minimize confounds, analysis of covariance (ANCOVA) may be used.

3. Implications and Conclusion

3.1. Implications

This research addresses current objectives for children's scholastic success in mathematics, which often extends to science, while at the same time considering the development of the whole child in order to foster these goals. If it can be shown that musical training enhances logical thinking, policy-makers may consider retaining or even increasing music education in schools as suggested before. What if students' spatial-temporal, pattern-recognition and problem-solving abilities are indeed improved via music lessons, which in turn support mathematical thinking? By seeking creative ways to help children learn and develop, we would be serving children's needs, academic and beyond, responsibly and potentially more successfully.

3.2. Conclusion

Both music and mathematics are a part of our individual and collective evolution. We begin to make sense of this world even in the womb via our mother's voice and respond particularly to her singing, regardless of culture (Nakata & Trehub, 2004). We also build neural networks that process the "statistical structure of experienced events" from birth (Goswami & Bryant, 2007). Hypotheses linking the two domains have existed for thousands of years (Burkert, 1972) and are reappearing now in the wake of new opportunities for studying the brain (Schlaug, 2001; Schlaug, Norton, Overy & Winner, 2004; Fujioka, Ross, Kakigi, Pantev, & Trainor, 2006; Posner, Rothbart, Sheese & Kieras, 2008).

This research seeks to build upon existing literature and gain concrete evidence through sound methodology in order to elucidate possible causality between musical training and mathematical thinking. More research is needed, but perhaps there is enough evidence in the meantime to strengthen the discourse regarding expansive ways to enhance the educational lives of children.

Acknowledgements

I would like to thank my supervisor at the University of Cambridge, Dr. Linda Hargreaves for her continued insight and patience. I would also like to thank the Faculty of Education at the university, my college Peterhouse and the Fund for Women Graduates for their assistance in funding my research.

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